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CALCULUS METHODS

integrals.theory.techniques.sequences

REVIEW OF BASIC CALCULUS FOR BUSINESS, BIOLOGY & PSYCHOLOGY MAJORS

LIMITS & CONTINUITY

- $\lim_{x \to a} f(x) = L$ if $f(x)$ **is close to** L for all x sufficiently close (but not equal) to a.
- $f(x)$ is continuous at $x = a$ if:
  1. $f(a)$ is defined,
  2. $\lim_{x \to a} f(x) = L$ exists, and
  3. $L = f(a)$

INTEGRALS

THE DEFINITE INTEGRAL

- **LET $f(x)$ BE CONTINUOUS ON $[a, b]$**
  1. **Riemann Sum Definition of Definite Integral**
     a. Divide $[a, b]$ into n equal subintervals of length $h = \frac{b-a}{n}$.
     b. Let $x_0 = a, x_1, x_2, \ldots, x_n = b$ denote the endpoints of the subintervals. They are found by: $x_0 = a$, $x_1 = a + h$, $x_2 = a + 2h$, $x_3 = a + 3h$, ..., $x_n = a + nh = b$.
     c. Let $m_1, m_2, \ldots m_n$ denote the midpoints of the subintervals. They are found by: $m_1 = 0.5(x_0 + x_1)$, $m_2 = 0.5(x_1 + x_2)$, $m_3 = 0.5(x_2 + x_3)$, ..., $m_n = 0.5(x_n - 1 + x_n)$.
     $\int_a^b f(x)dx \approx h[f(m_1) + f(m_2) + \ldots + f(m_n)]$.
  2. **Midpoint Rule:** $\int_a^b f(x)dx = \lim_{n \to \infty} h[f(x_1) + f(x_2) + \ldots + f(x_n)]$
  3. **Trapezoid Rule:** $\int_a^b f(x)dx \approx \frac{h}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-1}) + f(x_n)]$
  4. **Simpson's Rule:** $\int_a^b f(x)dx \approx \frac{h}{3}[f(x_0) + 4f(m_1) + 2f(x_1) + 4f(m_2) + 2f(x_2) + \ldots + 2f(x_{n-1}) + 4f(m_n) + f(x_n)]$

THE INDEFINITE INTEGRAL

- **$F(x)$ IS CALLED AN ANTIDERIVATIVE OF $f(x)$, IF $F'(x) = f(x)$**
  1. The most general antiderivative is denoted $\int f(x)dx$.
  2. $\int f(x)dx$ is also called the Indefinite Integral of $f(x)$.
- **Fundamental Theorem of Calculus**
  1. If $F'(x) = f(x)$ and $f(x)$ is continuous on $[a, b]$, then $\int_a^b f(x)dx = F(b) - F(a)$.

INTEGRATION FORMULAS

1. $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$
2. $\int kf(x)dx = k\int f(x)dx$ if k is a constant
3. $\int u^n du = \frac{u^{n+1}}{n+1} + C$
4. $\int \frac{1}{u} du = ln|u| + C$
5. $\int e^u du = e^u + C$
6. If $y = f(x) \geq 0$ on $[a, b]$, $\int_a^b f(x)dx$ gives the area under the curve.
7. If $f(x) \geq g(x)$ on $[a, b]$, $\int_a^b [f(x) - g(x)]dx$ gives the area between the two curves $y = f(x)$ and $y = g(x)$.
8. Average value of $f(x)$ on $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x)dx$.
9. Volume of the solid of revolution obtained by revolving about the x-axis the region under the curve $y = f(x)$ from $x = a$ to $x = b$ is $\int_a^b \pi[f(x)]^2 dx$.

INTEGRATION BY PARTS

1. Factor the integrand into two Parts: u and dv.
2. Find du and $v = \int dv$.
3. Find $\int v du$.
4. Set $\int u dv = uv - \int v du$.

INTEGRATION BY SUBSTITUTION

- **TO SOLVE $\int f(g(x))g'(x)dx$**
  1. Set $u = g(x)$, where $g(x)$ is chosen so as to simplify the integrand.
  2. Substitute $u = g(x)$ and $du = g'(x)dx$ into the integrand.
     a. This step usually requires multiplying or dividing by a constant.
  3. Solve $\int f(u)du = F(u) + C$.
  4. Substitute $u = g(x)$ to get the answer: $F(g(x)) + C$.

IMPROPER INTEGRALS

- **INFINITE LIMITS OF INTEGRATION**
  1. $\int_a^{\infty} f(x)dx = \lim_{b \to \infty} \int_a^b f(x)dx$
  2. $\int_{-\infty}^b f(x)dx = \lim_{a \to -\infty} \int_a^b f(x)dx$
- **IMPROPER AT THE LEFT OR RIGHT ENDPOINTS**
  1. If $f(x)$ is discontinuous at $x = b$, $\int_a^b f(x)dx = \lim_{h \to b^-} \int_a^h f(x)dx$.
  2. If $f(x)$ is discontinuous at $x = a$, $\int_a^b f(x)dx = \lim_{h \to \infty} \int_a^h f(x)dx$.

DERIVATIVES & THEIR APPLICATIONS

DERIVATIVE BASICS

- **DEFINITION OF DERIVATIVE**
  1. $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$
  2. If $y = f(x)$, the derivative $f'(x)$ is also denoted $\frac{dy}{dx}$.
- **FORMULAS:**
  1. **Power Rule:** $\frac{d}{dx}(x^n) = nx^{n-1}$
  2. $\frac{d}{dx}(e^{kx}) = ke^{kx}$
  3. $\frac{d}{dx}(lnx) = \frac{1}{x}$
  4. **General Power Rule:** $\frac{d}{dx}([f(x)]^n) = n[f(x)]^{n-1}f'(x)$
  5. $\frac{d}{dx}[e^{f(x)}] = e^{f(x)}f'(x)$
  6. $\frac{d}{dx}[lnf(x)] = \frac{f'(x)}{f(x)}$
  7. **Sum or Difference Rule:** $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$
  8. **Constant Multiple Rule:** $\frac{d}{dx}[kf(x)] = kf'(x)$
  9. **Product Rule:** $\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$
  10. **Quotient Rule:** $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$
  11. **Chain Rule:** $\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$, or $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$
  12. **Derivative of an inverse function:** $\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$

IMPLICIT DIFFERENTIATION

- **GIVEN AN EQUATION INVOLVING FUNCTION OF x AND y, TO FIND: $\frac{dy}{dx}$**
  1. Differentiate both sides of the equation with respect to x, treating y as a function of x and applying the chain rule to each term involving y (i.e. $\frac{d}{dx}[f(y)] = f'(y)\frac{dy}{dx}$).
  2. Move all terms with $\frac{dy}{dx}$ to left side and all other terms to the right.
  3. Solve for $\frac{dy}{dx}$.

CURVE SKETCHING

- **STEPS TO FOLLOW IN SKETCHING THE CURVE $y = f(x)$:**
  1. Determine the domain of $f(x)$.
  2. Analyze all points where $f(x)$ is discontinuous. Sketch the graph near all such points.
  3. Test for vertical, horizontal and oblique asymptotes.
     a. $f(x)$ has a vertical asymptote at $x = a$ if: $\lim_{x \to a^-} f(x) = \pm\infty$ or $\lim_{x \to a^+} f(x) = \pm\infty$.
     b. $f = (x)$ has a horizontal asymptote $y = b$ if: $\lim_{x \to \infty} f(x) = b$ or $\lim_{x \to -\infty} f(x) = b$.
     c. Sketch any asymptotes.
  4. Find $f'(x)$ and $f''(x)$.
  5. Find all critical points. These are points $x=a$ where $f'(a)$ does not exist or $f'(a)=0$. Repeat steps **5.a.** & **5.b.** for each critical point $x = a$:
     a. If $f(x)$ is continuous at $x = a$,
        i. $f(x)$ has a relative maximum at $x = a$ if:
           (a). $f'(a) = 0$ and $f''(a) < 0$, or
           (b). $f'(x) > 0$ to the left of a and $f'(x) < 0$ to the right of a.
        ii. $f(x)$ has a relative minimum at $x=a$ if:
           (a). $f'(a) = 0$ and $f''(a) > 0$, or
           (b). $f'(x) < 0$ to the left of a and $f'(x) > 0$ to the right of a.
     b. Sketch $f(x)$ near $(a, f(a))$.
  6. Find all possible inflection points. These are points $x = a$ where $f''(x)$ does not exist or $f''(x) = 0$. Repeat steps **6a.** & **6b.** for each such $x = a$:
     a. $f(x)$ has an inflection point at $x = a$ if $f(x)$ is continuous at $x = a$ and
        i. $f''(x) < 0$ to the left of a and $f''(x) > 0$ to the right of a, **or**
        ii. $f''(x) > 0$ to the left of a and $f''(x) < 0$ to the right of a.
     b. Sketch $f(x)$ near $(a, f(a))$.
  7. If possible, plot the x- and y- intercepts.
  8. Finish the sketch.

OPTIMIZATION PROBLEMS

- **TO OPTIMIZE SOME QUANTITY SUBJECT TO SOME CONSTRAINT:**
  1. Identify and label quantity to be maximized or minimized.
  2. Identify and label all other quantities.
  3. Write quantity to be optimized as a function of the other variables. This is called the objective function (or objective equation).
  4. If the objective function is a function of more than one variable, find a constraint equation relating the other variables.
  5. Use the constraint equation to write the objective function as a function of only one variable.
  6. Using the curve sketching techniques, locate the maximum or minimum of the objective function.

APPROXIMATIONS & DIFFERENTIALS

- **LET $y = f(x)$ AND ASSUME $f'(a)$ EXISTS**
  1. The Equation of the Tangent Line to $y = f(x)$ at the point $(a, f(a))$ is $y - f(a) = f'(a)(x-a)$.
  2. The differential of y is $dy = f'(x)dx$.
  3. Linear Approximation, or Approximation by Differentials. Set $dx = \Delta x = x - a$, $\Delta y = f(x) - f(a)$.
     The equation of the tangent line becomes:
     $\Delta y = f'(a)\Delta x = f'(a)dx$. If Δx is small, then $\Delta y \approx dy$.
     That is, $f(x) \approx f(a) + f'(a)(x - a)$.
  4. The nth Taylor polynomial of $f(x)$ centered at $x = a$ is $p_n(x) = f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!} + ... + \frac{f^{(n)}(a)(x-a)^n}{n!}$

MOTION

- **FORMULA**
  If $s = s(t)$ represents the position of an object at time t relative to some fixed point, then $v(t)=s'(t)$ = velocity at time t and $a(t) = v'(t) = s''(t)$ = acceleration at time t.

APPLICATIONS TO BUSINESS & ECONOMICS

COST, REVENUE & PROFIT

1. $C(x)$ = cost of producing x units of a product
2. $p = p(x)$ = price per unit; ($p = p(x)$ is also called the demand equation)
3. $R(x) = xp$ = revenue made by producing x units
4. $P(x) = R(x) - C(x)$ = profit made by producing x units
5. $C'(x)$ = marginal cost
6. $R'(x)$ = marginal revenue
7. $P'(x)$ = marginal profit

COMPOUNDING INTEREST

- **STARTING WITH A PRINCIPAL P_0**
  1. If the interest is compounded for t years with m periods per year at the interest rate of r per annum, the compounded amount is: $P = P_0(1 + \frac{r}{m})^{mt}$.
  2. If interest is continuously compounded, $m \to \infty$ and the formula becomes:
     $P = \lim_{m \to \infty} P_0(1 + \frac{r}{m})^{mt} = P = P_0e^{rt}$.
  3. The formula $P = P_0e^{rt}$ gives the value at the end of t years, assuming continuously compounded interest. P_0 is called the present value of P to be received in t years and is given by the formula $P_0 = Pe^{-rt}$.

ELASTICITY OF DEMAND

- **SOLVING FOR X IN THE DEMAND EQUATION $p = p(x)$ GIVES $x = f(p)$**
  1. Demand function which gives the quantity demanded x as a function of the price p.
  2. The elasticity of demand is:
     $$E(p) = \frac{-pf'(p)}{f(p)}.$$
- **DEMAND IS ELASTIC AT $p = p0$ IF $E(p0) > 1$**
  In this case, an increase in price corresponds to a decrease in revenue.
- **DEMAND IS INELASTIC AT $p = p0$ IF $E(p0) < 1$**
  In this case, an increase in price corresponds to an increase in revenue.

CONSUMERS' SURPLUS

- **IF A COMMODITY HAS DEMAND EQUATION $p = p(x)$**
  Consumers' Surplus is given by $\int_0^a [p(x) - p(a)]dx$ where a is the quantity demanded and $p(a)$ is the corresponding price.

EXPONENTIAL MODELS

EXPONENTIAL GROWTH & DECAY

- **EXPONENTIAL GROWTH: $y = P_0e^{kt}$**
  1. Satisfies the differential equation $y' = ky$
  2. P_0 is the initial size, $k > 0$ is called the growth constant.
  3. The time it takes for the size to double is given by: $\frac{ln2}{k}$.
- **EXPONENTIAL DECAY: $y = P_0e^{-\lambda t}$**
  1. Satisfies the differential equation $y' = -\lambda y$.
  2. P_0 is the initial size, $\lambda > 0$ is called the decay constant.
  3. The half life $t_{1/2}$ is the time it takes for y to become $P_0/2$. It is found by $t_{1/2} = \frac{ln.5}{-\lambda} = \frac{ln2}{\lambda}$.

OTHER GROWTH CURVES

- **THE LEARNING CURVE: $y = M(1-e^{-kt})$**
  Satisfies the differential equation $y' = k(M-y)$, $y(0) = 0$ where M and k are positive constants.
- **THE LOGISTIC GROWTH CURVE:**
  $y = \frac{M}{1+Be^{-Mkt}}$ satisfies the differential equation $y' = ky(M-y)$ where B, M and k are positive constants.

PROBABILITY

DEFINITIONS

- **PROBABILITY DENSITY FUNCTION** For the continuous random variable X is a function $p(x)$ satisfying: And $p(x) \geq 0$ if $A \leq x \leq b$ and $\int_A^B p(x)\,dx = 1$, where we assume the values of x lie in $[A, B]$.
- **THE PROBABILITY THAT** $a \leq X \leq b$ is $P[a \leq X \leq b] = \int_a^b p(x)\,dx$
- **EXPECTED VALUE, OR MEAN OF X** Given by $m = E(X) = \int_A^B xp(x)\,dx$
- **VARIANCE OF X:** Given by $\sigma^2 = var(X) = \int_A^B (x-\mu)^2 p(x)dx = \int_A^B x^2 p(x)dx - \mu^2$

COMMON PROBABILITY DENSITY FUNCTIONS

- **UNIFORM DISTRIBUTION FUNCTION:** $p(x) = \frac{1}{B-A}, \mu = E(X) = \frac{B+A}{2}, var(X) = \frac{(B-A)^2}{12}$
- **EXPONENTIAL DENSITY FUNCTION:** $p(x) = \lambda e^{-\lambda x}$. In this case, $A = 0$, $B = \infty$, $\mu = E(X) = 1/\lambda$, $var(X) = 1/\lambda^2$.
- **NORMAL DENSITY FUNCTION:** with $E(X) = \mu$ and $var(X) = \sigma^2$ is:

$$p(x)\frac{1}{\sqrt{2\pi}\,\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi}\,\sigma}exp\left[\frac{(x-\mu)^2}{2s^2}\right]$$

CALCULUS OF FUNCTIONS OF TWO VARIABLES

PARTIAL DERIVATIVES

- **WHERE $f(x, y)$ IS A FUNCTION OF TWO VARIABLES x AND y**
  1. $\frac{\partial f}{\partial x}$ is the derivative of $f(x,y)$ with respect to x, treating y as a constant.
  2. $\frac{\partial f}{\partial y}$ is the derivative of $f(x,y)$ with respect to y, treating x as a constant.
  3. $\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial y} - \frac{\partial f}{\partial x}$ is the second partial derivative of $f(x,y)$ with respect to x twice, keeping y constant each time.
  4. $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial y}\frac{\partial f}{\partial x}$ is the second partial derivative of $f(x,y)$, first with respect to x keeping y constant, then with respect to y keeping x constant.
  5. Other notation for partial derivatives: $f_x(x,y) = \frac{\partial f}{\partial x}, f_{xx}(x,y) = \frac{\partial^2 f}{\partial x^2}, f_{xy}(x,y) = \frac{\partial^2 f}{\partial y \partial x}$.

DIFFERENTIALS

- If $f = f(x, y)$
  1. $df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = f_x(x,y)dx + f_y(x,y)dy$
  2. Setting $dx = \Delta x = x - a$, $dy = \Delta y = y - b$ and $\Delta f = f(x,y) - f(a,b)$, if Δx and Δy are both small, then $\Delta f \approx df$. That is: $f(x,y) \approx f(a,b) + f_x(a,b)\Delta x + f_y(a,b)\Delta y$.

RELATIVE EXTREMA TEST

- **TO LOCATE RELATIVE MAXIMA, RELATIVE MINIMA AND SADDLE POINTS ON THE GRAPH OF $z = f(x,y)$.**
  1. Solve simultaneously: $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$. For each ordered pair (a, b) such that $\frac{\partial f}{\partial x}(a,b) = 0$ and $\frac{\partial f}{\partial y}(a,b) = 0$, apply the following test.
  2. Set $A = \frac{\partial^2 f}{\partial x^2}(a,b)$, $B = \frac{\partial^2 f}{\partial y^2}(a,b)$, $C = \frac{\partial^2 f}{\partial x \partial y}(a,b)$ and $D = AB - C^2$.
     a. If $D > 0$ and $A > 0$, then $f(x,y)$ has a relative minimum at (a,b).
     b. If $D > 0$ and $A < 0$, then $f(x,y)$ has a relative maximum at (a,b).
     c. If $D < 0$, then $f(x,y)$ has a saddle point at (a,b).
     d. If $D = 0$, then the test fails. $f(x,y)$ may or may not have an extremum or saddle point at (a,b).

THE METHOD OF LAGRANGE MULTIPLIERS

- **SOLVES CONSTRAINED OPTIMIZATION PROBLEMS. TO MAXIMIZE OR MINIMIZE $f(x,y)$ SUBJECT TO THE CONSTRAINT $g(x,y)=0$**
  1. Define the new function $F(x, y, \lambda) = f(x,y) + \lambda g(x,y)$.
  2. Solve the system of three equations:
     a. $\frac{\partial F}{\partial x} = 0$,
     b. $\frac{\partial F}{\partial y} = 0$, and
     c. $\frac{\partial F}{\partial \lambda} = 0$ simultaneously.

This is usually accomplished in four steps:

Step 1: Solve **a.** and **b.** for λ and equate the solutions.

Step 2: Solve the resulting equation for one of the variables, x or y.

Step 3: Substitute this expression for x or y into equation **c.** and solve the resulting equation of one variable for the other variable.

Step 4: Substitute the value found in Step 3 into the equation found in Step 2. Use one of the equations from Step 1 to find λ. This gives the value of x and y.

DOUBLE INTEGRALS

1. If R is the region in the plane bounded by the two curves $y = g(x)$, $y = h(x)$ and the two vertical lines $x = a$, $x = b$, then the double integral $\iint_R f(x,y)\,dxdy$ is equal to the iterated integral $\int_a^b \left(\int_{g(x)}^{h(x)} f(x,y)\,dy \right) dx$.
2. To evaluate the iterated integral $I = \int_a^b \left(\int_{g(x)}^{h(x)} f(x,y)\,dy \right) dx$
   a. find an antiderivative $F(x,y)$ for $f(x,y)$ with respect to y keeping x constant. That is: $\frac{\partial F}{\partial y} = f(x,y)$.
   b. Set: $I = \int_a^b [F(x,h(x)) - F(x,g(x))]\,dx$.
   c. Solve this integral. The integrand is a function of one variable.

DIFFERENTIAL EQUATIONS

- **A DIFFERENTIAL EQUATION IS: Any equation involving a derivative.** For example, it could be an equation involving $\frac{dy}{dx}$ (or y', or $y'(x)$), y and x.
- **A SOLUTION IS:** A function $y = y(x)$, such that $\frac{dy}{dx}$, y and x satisfy the original equation.
- **AN INITIAL VALUE PROBLEM** also specifies the value of the solution $y(a)$ at some point $x = a$
- **SIMPLE DIFFERENTIAL EQUATIONS** can be solved by separation of variables and integration. For example, the equation $f(x) = g(y)\frac{dy}{dx}$ can be written as $f(x)dx = g(y)dy$ and can be solved by integrating both sides: $\int f(x) = \int g(y)\frac{dy}{dx}$.

FORMULAS FROM PRE-CALCULUS

LOGARITHMS & EXPONENTIALS

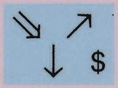
1. $y = ln x$ if and only if $x = e^y = \exp(y)$
2. $ln e^x = x$
3. $e^{ln\,x} = x$
4. $e^x e^y = e^{x+y}$
5. $\frac{e^x}{e^y} = e^{x-y}$
6. $(e^x)^y = e^{xy}$
7. $e^0 = 1$
8. $ln(xy) = ln\,x + ln\,y$
9. $ln(x/y) = ln\,x - ln\,y$
10. $ln(x^y) = y\,ln\,x$
11. $ln\,1 = 0$
12. $ln\,e = 1$

ALGEBRAIC FORMULAS

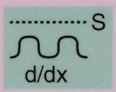
1. If $a \neq 0$, the solutions to $ax^2 + bx + c = 0$ are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
2. Point-slope equation of a line: $y - y_0 = m(x - x_0)$.

DEVELOP A PROBLEM SOLVING STRATEGY

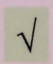
- **Three key issues**
  1. Understand the business or scientific principle required to solve the problem.
  2. Develop a correct mathematical strategy.
  3. Logically approach solving the problem.
- **Eight useful steps in problem solving:**
  1. **Prepare a rough sketch** or diagram based on the subject of the problem. For business applications, set this up using business terms; for science, use physical variables.



  2. **Identify all relevant variables**, concepts and constants.
     Note: Do not simply search for the "right" equation in your notes or text. You may have to select your own variables to solve the problem.
  3. **Describe the problem** using appropriate mathematical relationships or graphs.
  4. **Obtain any constants** from the stated problem or textbook. Make sure you have all the essential data.
     Hint: You may have extra information.



  5. **The hard part:** Derive a mathematical expression for the problem. Make sure that the equation, constants and data give the right unit for the final answer.
  6. **Carry out the appropriate mathematical manipulation**, differentiate, integrate, find limits, etc.
  7. **The easy part**: Plug numbers into the equation. Obtain a quick approximate answer, then use a calculator to obtain an exact numerical answer.



  8. **Check the final answer** using the original statement of the problem, your sketch and common sense; are the units correct? Sign? Magnitude?

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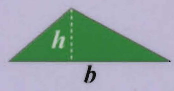
GEOMETRIC FORMULAS

Perimeter: The perimeter, P, of a two-dimensional shape is the sum of all side lengths.
Area: The area, A, of a two dimensional shape is the number of square units that can be put in the region enclosed by the sides. *Note: Area is obtained through some combination of multiplying heights and bases, which always form 90° angles with each other, except in circles.*
Volume: The volume, V, of a three-dimensional shape is the number of cubic units that can be put in the region enclosed by all the sides.

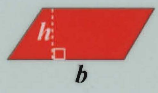
Square Area:
$A = b^2$; If $b = 8$, then: $A = 64$ square units.

Rectangle Area:
$A = hb$, or $A = lw$; If $h = 4$ and $b = 12$, then:
$A = (4)(12)$, $A = 48$ square units.

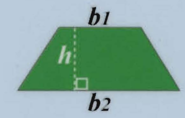
Triangle Area:
$A = \frac{1}{2}bh$; If $h = 8$ and $b = 12$, then:
$A = \frac{1}{2}(8)(12)$, $A = 48$ square units.



Parallelogram Area:
$A = hb$; If $h = 6$ and $b = 9$, then:
$A = (6)(9)$, $A = 54$ square units.



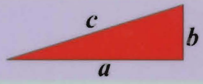
Trapezoid Area:
$A = \frac{1}{2}h(b1 + b2)$; If $h = 9$, $b1 = 8$ and $b2 = 12$, then:
$A = \frac{1}{2}(9)(8 + 12)$, $A = \frac{1}{2}(9)(20)$, $A = 90$ square units.



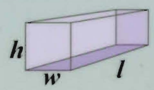
Circle Area:
$A = \pi r^2$; If $r = 5$, then:
$A = \pi(5)^2 = (3.14)25 = 78.5$ square units.

Circumference: $C = 2\pi r$; If $r = 5$, then:
$C = (2)(\pi)(5) = 10(3.14) = 31.4$ units.

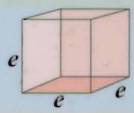
Pythagorean Theorem:
If a right triangle has hypotenuse c and sides a and b, then: $c^2 = a^2 + b^2$.



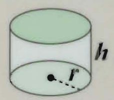
Rectangular Prism Volume:
$V = lwh$; If $l = 12$, $w = 3$ and $h = 4$, then:
$V = (12)(3)(4)$, $V = 144$ cubic units.



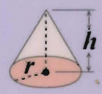
Cube Volume:
$V = e^3$; each edge length, e, is equal to the other edge in a cube.
If $e = 8$, then: $V = (8)(8)(8)$, $V = 512$ cubic units.



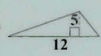
Cylinder Volume:
$V = \pi r^2 h$; If radius $r = 9$ and $h = 8$, then:
$V = \pi(9)^2(8)$, $V = (3.14)(81)(8)$, $V = 2034.72$ cubic units.



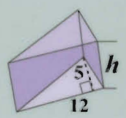
Cone Volume:
$V = \frac{1}{3}\pi r^2 h$; If $r = 6$ and $h = 8$, then:
$V = \frac{1}{3}\pi(6)^2(8)$, $V = \frac{1}{3}(3.14)(36)(8)$, $V = 301.44$ cubic units.



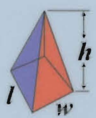
Triangular Prism Volume:
$V =$ (area of triangle)h; If



has an area equal to $\frac{1}{2}(5)(12)$, then:
$V = 30h$ and if $h = 8$, then: $V = (30)(8)$, $V = 240$ cubic units.



Rectangular Pyramid Volume:
$V = \frac{1}{3}$(area of rectangle)h; If $l = 5$ and $w = 4$ the rectangle has an area of 20, then: $V = \frac{1}{3}(20)h$ and if $h = 9$, then:
$V = \frac{1}{3}(20)(9)$, $V = 60$ cubic units.



Sphere Volume:
$V = \frac{4}{3}\pi r^3$; If radius $r = 5$, then: $V = \frac{4}{3}(3.14)(5)^3$, $V = 523.3$ cubic units.